Quiz 1 Solutions

Your name here:

1 Analyze this, dimensionally

Energy is required to assemble charged particles into a sphere. It is reasonable to assume that the energy required E ($[E] = \mathrm{J}$) might depend on the total amount of charge Q ($[Q] = \mathrm{C}$), the size of the sphere R ($[R] = \mathrm{m}$), and the coulomb constant k ($[k] = \frac{\mathrm{Nm}^2}{\mathrm{C}^2}$).

1. (4 pt) Use dimensional analysis to find an approximate formula for E in terms of the other variables given.

   $$[E] = \mathrm{J} = \mathrm{N} \cdot \mathrm{m}$$

   Now I pick k to do first because it has unique units and is complicated, so I want to make $[E]$ look like $[k]$

   $$[E] = \mathrm{N} \cdot \mathrm{m} = \mathrm{N} \cdot \mathrm{m} \cdot \frac{\mathrm{m}}{\mathrm{m}} \cdot \frac{\mathrm{C}^2}{\mathrm{C}^2} = \frac{\mathrm{Nm}^2}{\mathrm{C}^2} \cdot \frac{\mathrm{C}^2}{\mathrm{m}}$$

   Now I can plug in $[k]$ into $[E]$

   $$[E] = [k] \cdot \frac{\mathrm{C}^2}{\mathrm{m}}$$

   Fortuitously, we immediately recognize that $\mathrm{C}^2 = [\mathrm{Q}^2]$ and $\frac{1}{\mathrm{m}} = \frac{1}{[\mathrm{R}]}$

   $$[E] = \frac{[k][Q^2]}{[R]}$$

   or

   $$E \sim \frac{kQ^2}{R}$$

2. (1 pt) Evaluate your expression for a sphere with radius 10 cm with a total charge of 10 C. The numerical value of the coulomb constant in SI units is $k = 9 \times 10^9 \ \frac{\mathrm{Nm}^2}{\mathrm{C}^2}$

   First we convert 10 cm to m and get $R = 10^{-1}$ m

   $$E \approx \frac{(9 \times 10^9 \ \frac{\mathrm{Nm}^2}{\mathrm{C}^2})(10 \ \mathrm{C})^2}{10^{-1}\mathrm{m}}$$

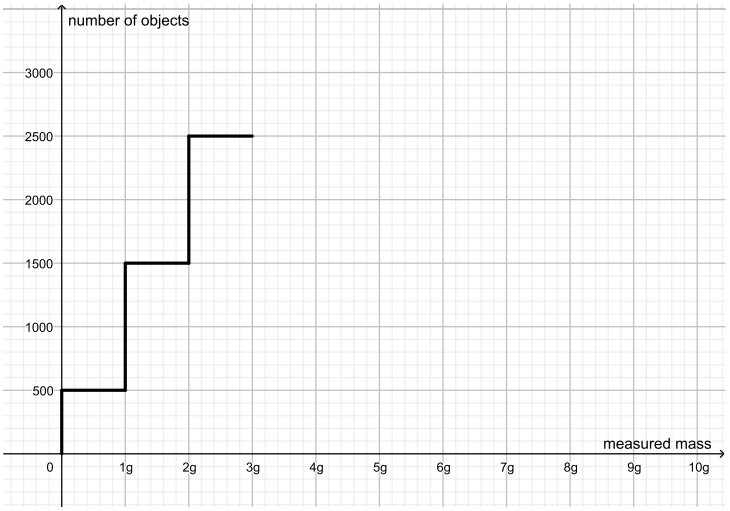
   $$E \approx 9 \times 10^{9+2+1} \ \mathrm{J}$$

   $$\boxed{E \approx 9 \times 10^{12} \ \mathrm{J}}$$

1.1 Discrete means either right or wrong

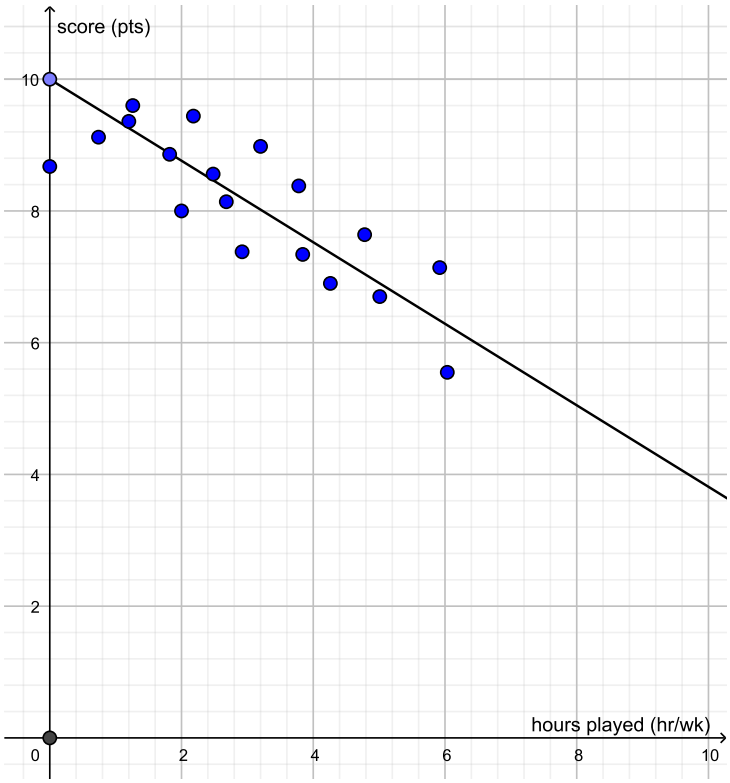
You are attempting to measure grains of sand with mass of exactly .001g each using a scale that can only read out in 1g increments (eg it can display either 1 g or 2 g, but not 1.5 g), but is perfectly accurate and rounds correctly.

1. *Draw the graph of the actual number of grains on the scale as a function of the measured mass.*



1.2 Using trend lines

Below is an (entirely made up) graph relating number of hours spent per week playing fortnite and the student's score out of 10 points on an assessment. The points are free of error.



1. *Plot the trend line*

2. *Determine the expected number of hours that a student spent playing forntnite in a week if there score on the exam was 20% and comment on whether this extrapolation was reasonable in a real dataset.*
   First we get the equation of the line in $y = mx + b$ form.
   b is about 10 points
   Now choosing points (0,10) and (6.4,6) we can calulate the slope $m = \frac{\Delta y}{\Delta x} = \frac{(6-10)\text{pts}}{(6.4-0)\frac{\text{hr}}{\text{wk}}}$
   so

$$m \approx -\frac{2}{3}\frac{\text{pts}}{\frac{\text{hr}}{\text{wk}}}$$

   Now we plug in values (dropping units temporarily for clarity)

$$2 = -\frac{2}{3}r + 10$$

$$\boxed{r = 12\frac{\text{hr}}{\text{wk}}}$$

2 Vectors

Below are the directions a new friend gave you to find a certain point in an empty field.

First walk $\vec{q} = -2\hat{x} - \hat{y}$
From there, walk $\vec{s} = \hat{x} - 3\hat{y}$
From there, walk $\vec{u} = \hat{x} + 2\hat{y}$
Finally walk $\vec{w} = 3\hat{x} + \hat{y}$

1. *Write the displacement to that point as a vector in terms of* $\hat{x}$ *and* $\hat{y}$*.*

   To get the displacement we just separately add up all the x displacements and the y displacements.
   $\Delta\vec{r} = (-2 + 1 + 1 + 3)\hat{x} + (-1 - 3 + 2 + 1)\hat{y}$

   $$\boxed{\Delta\vec{r} = 3\hat{x} - \hat{y}}$$

2. *What was the total distance traveled?*

   We have to add up all the distances for each of the vectors we get these by taking the magnitude of the vectors.
   $|\vec{q}| = q = \sqrt{2^2 + 1^2} = \sqrt{5}$
   $s = \sqrt{10}$
   $u = \sqrt{5}$
   $w = \sqrt{10}$
   So the total distance traveled, d, is

   $$\boxed{d = 2\sqrt{5} + 2\sqrt{10}}$$