AP Quiz 1: Mathematical Tools

1 Analyze this, dimensionally

(5 pt) Use dimensional analysis to find a formula that relates the power given off by a star (P, units $\mathrm{W} = \frac{\mathrm{J}}{\mathrm{s}}$) with its temperature ($T$), its surface area, A and a constant σ, which has units of $\frac{\mathrm{kg}}{\mathrm{s}^3 \cdot \mathrm{K}^4}$

Start with the units of everything
$[P] = \frac{\mathrm{J}}{\mathrm{s}}$
$[T] = \mathrm{K}$
$[A] = \mathrm{m}^2$
$[\sigma] = \frac{kg}{s^3 \cdot K^4}$

I could either start by breaking J in $[P]$ into base units. It is more efficient to simplify $[\sigma]$.
Note that $\frac{\mathrm{J}}{\mathrm{s}} = \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^3}$ which means that
$[\sigma] = \frac{\mathrm{J}}{\mathrm{s} \cdot \mathrm{m}^2 \cdot \mathrm{K}^4}$
which I can solve to get
$\frac{\mathrm{J}}{\mathrm{s}} = \mathrm{m}^2 \cdot \mathrm{K}^4 [\sigma]$
substituting that into $[P]$ gives
$[P] = \mathrm{m}^2 \cdot \mathrm{K}^4 [\sigma]$
Now we use the other equations to subsitute for the remaining units to get

$$[P] = [A][T]^4[\sigma]$$

which means that

$$\boxed{P = \epsilon \sigma A T^4}$$

2 This problem is a relative and absolute mess

(5 pt) You are attempting to determine the volume of a chemically pure solid cube. You know the density is ρ exactly. You can either measure the mass m using a scale with an uncertainty of Δm or measure the side lengths L using a ruler with an uncertainty of ΔL and multiply to get volume. How big does Δm need to be before measuring the side lengths is the better choice? Answer in terms of other variables from the problem. You may assume that the errors involved are small enough that $(\Delta L)^2 \approx 0$ and $(\Delta m)^2 \approx 0$.

$$V + \Delta V = (L + \Delta L)^3$$
$$V + \Delta V \approx L^3 + 3L^2 \Delta L$$
$$V + \Delta V = V + 3\frac{V}{L}\Delta L$$
$$\boxed{\frac{\Delta V}{V} = 3\frac{\Delta L}{L}}$$
$$M = \rho V$$
$$M + \Delta M = \rho(V + \Delta V)$$
$$\frac{\Delta M}{M} = \frac{\Delta V}{V}$$

Combining this with the boxed equation above gives

$$\boxed{\frac{\Delta M}{M} = 3\frac{\Delta L}{L}}$$

If you chose to express this as absolute error, you got

$$\boxed{\Delta M = 3\frac{M}{L}\Delta L = 3\rho L^2 \Delta L}$$

3 A curved problem

Jerk is the term that physicists give to the rate of change of acceleration (like acceleration is the rate of change of velocity). The rate of change of jerk is termed "snap". The relationship between snap and displacement is $\Delta\vec{r} = \frac{1}{24}\vec{s}t^4$. You have a machine that always has some constant, unknown, snap ($\vec{s}$). You measure the displacement of an object as it moves and get the following data.

Snap and Displacement

t (s)	Δr (m)
1	2.02
1.1	3.05
1.2	4.23
1.3	6.46
1.4	7.84
1.5	9.91
1.6	12.29
1.7	17.92

1. *(3 pt) Linearize the equation. Be clear about your choice of variables and their units.*

$$x : \frac{1}{24}t^4$$

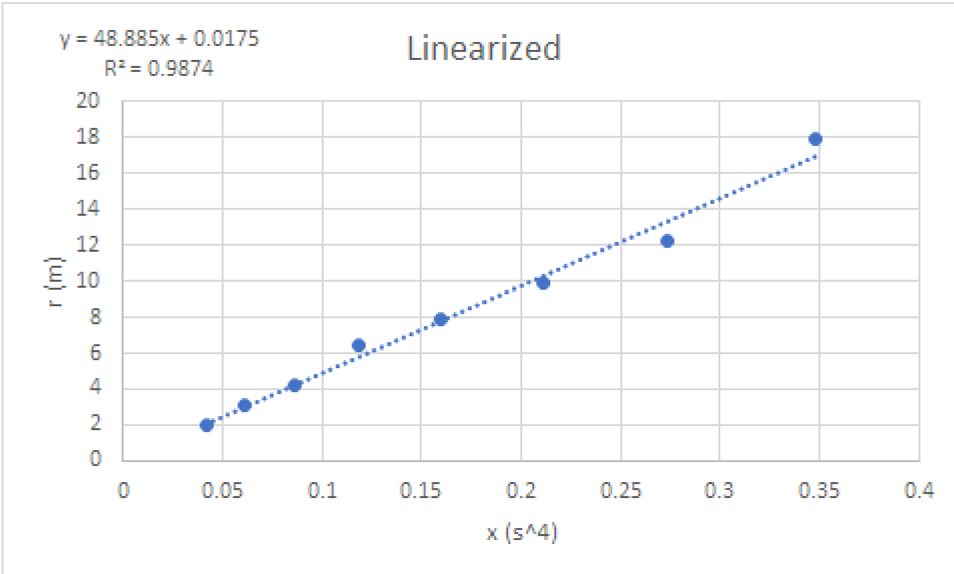
$$y : \Delta r$$

$$[x] = \mathrm{s}^4$$

$$[y] = \mathrm{m}$$

t	x	r
1	0.041667	2.020833
1.1	0.061004	3.050208
1.2	0.0864	4.2336
1.3	0.119004	6.426225
1.4	0.160067	7.843267
1.5	0.210938	9.914063
1.6	0.273067	12.288
1.7	0.348004	17.92221

2. *(2 pt) Plot the linearized data, find the trend line, and determine $\vec{s}$. I am fine with you doing the plot and trend-line on your calculator. In this case your score will be all or nothing based on whether the line and slope are correct. If you plot it by hand, partial credit may be possible.*



From which I get that the slope s is

$$\boxed{s = 48.9\ \frac{\mathrm{m}}{\mathrm{s}^4}}$$

4 Relative velocities

The velocity of object A relative to object B can be found by subtracting the velocity of object B from the velocity of object A (mathematically $\vec{v}_{ab} = \vec{v}_a - \vec{v}_b$). The velocity of object A relative to object B is $\vec{v}_{ab} = 10\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{x}} + 5\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{y}}$ and the velocity of object A relative to the ground is $\vec{v}_a = 5\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{x}} - 5\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{y}}$.

1. *(2 pt) What is the velocity of object B relative to the ground?*

   A. $\vec{v}_b = 15\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{x}}$

   B. $\vec{v}_b = 5\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{x}} + 10\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{y}}$

   $\boxed{\text{C.}}$ $\vec{v}_b = -5\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{x}} - 10\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{y}}$

   D. $\vec{v}_b = 10\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{x}} - 15\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{y}}$

   E. $\vec{v}_b = 5\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{x}} - 5\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{y}}$

   We are given $\vec{v}_a$ and $\vec{v}_{ab}$ and asked to solve for $\vec{v}_b$ so we solve the equation to get

$$\vec{v}_b = \vec{v}_a - \vec{v}_{ab}$$

   Then we just substract the vectors as normal.

2. *(3 pt) Object A measures the speed of object B using a radar gun. What value will the measurement have?*
   Speed is the magnitude of velocity, so we take

$$v_{ab} = \sqrt{\vec{v}_{ab} \cdot \vec{v}_{ab}}$$

$$v_{ab} = \sqrt{100 + 25}$$

$$\boxed{v_{ab} = 5\sqrt{5}\ \frac{\mathrm{m}}{\mathrm{s}}}$$

5 Something Extra to do

Check that your name is on your paper. Once finished, check again. Then compose a poem about the adventures you will have in school this year. (1 pt extra credit is possible here, but grading will be arbitrary and capricious. Also it will only be graded if all other problems show some earnest attempt at a solution)