Quiz: Basic Kinematics

For all problems, assume we are on Earth, assume gravity acts in the -ẑ direction, and neglect air resistance unless otherwise specified.

1 Launching the ball

1. *(4 pt) You drop a ball onto a surface that causes the ball to bounce back with 75% of the kinetic energy that it started with. If the ball is first dropped from a height of h_1, how long will the ball spend in the air between the first and second bounce?*

   Let's start with energy conservation...

$$\begin{aligned} \Delta E &= 0 \\ PE_i &= KE_f \\ mgh_i &= \frac{1}{2}mv^2 = KE_1 \end{aligned}$$

   Since the ball bounces back with 75% of it's kinetic energy, it's potential energy for the next bounce will also change accordingly.

$$\begin{aligned} mgh_2 &= 0.75KE_1 \\ mgh_2 &= 0.75mgh_1 \\ h_2 &= 0.75h_1 \end{aligned}$$

   Using kinematics we know that $\Delta y = \frac{1}{2}at^2$. Let's apply this principle to our situation.

$$\begin{aligned} \Delta h_2 &= \frac{1}{2}gt^2 \\ 0.75h_1 &= \frac{1}{2}gt^2 \end{aligned}$$

   Now we solve for time t.

$$\begin{aligned} t &= \sqrt{\frac{3h_1}{2g}} \quad \text{(to reach } h_2\text{)} \\ 2t &= \sqrt{\frac{6h_1}{g}} \quad \text{(time between bounce 1 and 2)} \end{aligned}$$

   As a result, the answer is $\boxed{\text{D.}}$

2 Shoot first, ask questions later

1. *(4 pt) You shoot a projectile horizontally from a height h above the ground with a speed v_0. The projectile hits the ground just as it reaches the target. What is the distance to the target?* Let's split movement of the projectile into x̂ and ŷ direction

$$\begin{aligned} \Delta x &= V_0 t \\ \Delta y &= \frac{1}{2}gt^2 \end{aligned}$$

   In order to find the distance to the target, we need to solve for Δx. To do so, we need to find the time the projectile spent in the air. We can get this from the Δy function!

$$\begin{aligned} \Delta y &= \frac{1}{2}gt^2 \quad \text{(note that } \Delta y \text{ is } h\text{)} \\ t &= \sqrt{\frac{2h}{g}} \end{aligned}$$

Now plug t into Δx function.

$$\Delta x = V_0 * \sqrt{\frac{2h}{g}} = d$$

As a result, the answer is $\boxed{\text{C.}}$

2. *(2 pt) In the same scenario as above, if air resistance were present, would the projectile need to be launched at an angle above or below the horizontal in order for it to hit as indicated? Consider only drag force, not less common effects.*

   $\boxed{\text{A.}}$ The drag force acts tangentially to the motion of the projectile. This means that it effects both $\hat{z}$ and $\hat{x}$ momentum, but it effects $\hat{x}$ momentum greater. This is because there is a greater velocity in the $\hat{x}$ direction and nearly no velocity in the $\hat{z}$ direction.

3 Sometimes you crash and burn

1. *(5 pt) Two objects have positions given by $\vec{r_1} = (2m + 1\frac{m}{s}t)\hat{x} + (5m + 2\frac{m}{s}t)\hat{y}$. The other object has a position given by $\vec{r_2} = (4m - 2\frac{m}{s}t)\hat{x} + (5m + 1\frac{m}{s}t)\hat{y}$. Will the object collide, and if so, at what time?*

   Let's find the time of collision for the objects in both the $\hat{x}$ and $\hat{y}$ direction. If the times are equivalent, then the objects will collide!
   First the $\hat{x}$:

$$\begin{aligned} 2 + 1t_x &= 4 - 2t_x \\ 3t_x &= 2 \\ t_x &= \frac{2}{3}s \end{aligned}$$

   Now for the $\hat{y}$:

$$\begin{aligned} 5 + 2t_y &= 5 + 1t_y \\ t_y &= 0s \end{aligned}$$

   Since $t_x \neq t_y$, **the objects will not collide**.

4 A throwback to 7th grade

1. *(5 pt, **AP**) Having forgotten the lessons of previous years, you challenge physics unicorn to a race and wage one million dollars. You move with a velocity given by $\vec{v} = v_s\hat{x}$. The physics unicorn moves with an acceleration given by $a_u\hat{x}$. If the physics unicorn finishes in a fraction $f << 1$ of your time, what was the distance from the starting point to the finish?*

   The time of the unicorn t_u is a fraction of the time you spend t_s, so $t_u = f t_s$.
   Since both you and the physics unicorn travel the same displacement Δx, we can find the time you spend moving.

$$\begin{aligned} v_s t_s &= \frac{1}{2}a_u(f t_s)^2 \\ t_s &= \frac{2v_s}{a_u f^2} \end{aligned}$$

We can plug t_s back into the equation for displacement of you $\Delta x = v_s t_s$ to find the distance from the starting point.

$$\boxed{\Delta x = \frac{2v_s^2}{a_u f^2}}$$

2. *(5 pt,* ***Honors****) Super-cactus challenged the physics unicorn to a race. Super cactus moves with constant velocity v_c and the physics unicorn moves with constant acceleration a_u. If the race ends with a tie, what was the distance from start to finish?* Because the race ends in a tie, the time the cactus moves t_c equals the time the unicorn moves t_u.

   Since they both travel the same displacement, we can equate their displacement equations.

$$\begin{aligned} v_c t &= \frac{1}{2} a_u (t)^2 \\ t &= \frac{2v_c}{a_u} \end{aligned}$$

   Plugging the time back into the displacement equation of the cactus, we get: $\boxed{\Delta x = \frac{2v_c^2}{a_u}}$