AP Quiz 2: Mathematical Kinematics

Your name-period here:

0 Constants

- $g = 10\ \frac{\text{m}}{\text{s}^2}$

1 Dropping the ball, twice

1. *You drop a ball on earth and then drop* ***a different ball*** *on an unknown planet in another star's orbit. Each time you drop the ball exactly the same unknown distance. On Earth it takes exactly* 1s *to hit the ground. On the unknown planet it takes exactly* 2s *to hit the ground. Below is a table of the surface acceleration of each planet.*

   - Planet A: $2\frac{\text{m}}{\text{s}^2}$
   - Planet B: $2.5\frac{\text{m}}{\text{s}^2}$
   - Planet C: $4\ \frac{\text{m}}{\text{s}^2}$
   - Planet D: $5\frac{\text{m}}{\text{s}^2}$

   (a) (2.5 pt) *Which planet are you on?*

   A. A
   $\boxed{\text{B.}}$ B
   C. C
   D. D
   E. There is no way to determine this with the given information

   We know that the distance we drop it is the same in both cases. Let's call it h.
   For Earth

   $$h = \frac{1}{2} g_e t_e^2$$

   and for the other planet

   $$h = \frac{1}{2} g_p t_p^2$$

   since h is the same

   $$\frac{1}{2} g_e t_e^2 = \frac{1}{2} g_p t_p^2$$

   Or, simplifying

   $$g_p = \left(\frac{t_e}{t_p}\right)^2 g_e$$

   We know the times, and that $g_e = 10\ \frac{\text{m}}{\text{s}^2}$, so

   $$g_p = \left(\frac{1}{2}\right)^2 \left(10\ \frac{\text{m}}{\text{s}^2}\right)$$

   $$g_p = 2.5 \frac{\text{m}}{\text{s}^2}$$

   (b) (2.5 pt) *What is the ratio of the masses of the balls?*

   A. 1:1
   B. 3:1
   C. 9:1
   D. 27:1

$\boxed{\text{E.}}$ There is no way to determine this with the given information

Gravitational acceleration is independent of mass, so we can't use it to find the object's masses.

(0pt) Does your name and period appear at the top of the paper? If not, explain why limerick that includes your name and the word "beagle."

2 Braking or breaking

1. (5pt) *A ship is traveling* 30 $\frac{\text{m}}{\text{s}}\hat{x}$ *when it spots a lighthouse 200 m ahead. If it can accelerate at* $-3\ \frac{\text{m}}{\text{s}^2}\hat{x}$*, does the ship hit the lighthouse?*

   We don't have time given, so we start by determining that using the velocity equation. We are in 1-D, so the equation is:

$$v_f = v_i + at$$

   with $v_f = 0$ because we want to stop.

   so

$$t = \frac{v_i}{-a} = 10 \text{ s}$$

   Now we use the displacement equation

$$\Delta x = v_i t + \frac{1}{2} a_i t^2$$

   With $t = 10$ s to see if we hit the animal.

$$\Delta x = 300 \text{ m} + \frac{1}{2}\left(-300 \text{ m}\right)$$

$$\Delta x = 150 \text{ m} < 200 \text{ m}$$

$$\boxed{\text{no, there is no collision.}}$$

3 Two blind mice

Two mice are running across an open field. One of them has an initial position vector of

$$\vec{r}_{1,0} = 10 \text{ m}\hat{\text{x}} - 6 \text{ m}\hat{\text{y}}$$

and a velocity of

$$\vec{v}_1 = -8 \ \frac{\text{m}}{\text{s}}\hat{\text{x}} + 4 \ \frac{\text{m}}{\text{s}}\hat{\text{y}}$$

The other has a position vector of

$$\vec{r}_{2,0} = -18 \text{ m}\hat{\text{x}} + 10 \text{ m}\hat{\text{y}}$$

and a velocity of

$$\vec{v}_2 = 6 \ \frac{\text{m}}{\text{s}}\hat{\text{x}} - 4 \ \frac{\text{m}}{\text{s}}\hat{\text{y}}$$

1. (5pt) Do they collide? If so, at what time? Show all work! *(5pt) Do they collide? If so, at what time? Show all work!*

   The general equation for the position vector with 0 acceleration is:

   $$\vec{r} = \vec{r}_0 + \vec{v}t$$

   In our cases that is

   $$\vec{r}_1 = \left(10 \text{ m} - 8 \ \frac{\text{m}}{\text{s}}t\right)\hat{\text{x}} + \left(-6 \text{ m} + 4 \ \frac{\text{m}}{\text{s}}\text{t}\right)\hat{\text{y}}$$
   $$\vec{r}_2 = \left(-18 \text{ m} + 6 \ \frac{\text{m}}{\text{s}}t\right)\hat{\text{x}} + \left(10 \text{ m} - 4 \ \frac{\text{m}}{\text{s}}\text{t}\right)\hat{\text{y}}$$

   We equate the x and y components of the two position vectors independently to get (dropping units and dividing out an unnecessary factor of 2)

   $$\hat{x} : 5 - 4t_x = -9 + 3t_x$$
   $$\hat{y} : -3 + 2t_y = 5 - 2t_y$$

   Solving them gives

   $$t_x = 2 \text{ s}$$
   $$t_y = 2 \text{ s}$$

   So:

   $$\boxed{\text{yes, } \ t = 2 \text{ s}}$$

4 Woolley's nightmare

1. *(4pt) A fighter pilot is flying at a constant velocity of* $300\ \frac{\mathrm{m}}{\mathrm{s}}\hat{\mathrm{x}}$ *when an unseen aircraft* 500 m *behind the first, and traveling at the same velocity, fires a short-range missile at the first aircraft. The missile's acceleration is* $10\ \frac{\mathrm{m}}{\mathrm{s}^2}\hat{\mathrm{x}}$ *and the missile's maximum range when fired from a stationary platform is* 1000 m. *Does the missile hit the aircraft before running out of fuel? If so, at what time? Ignore air resistance and show all work.*

First we move to the reference frame of the aircraft that is about to fire the missile to eliminate the initial velocities. Since the missile will start with the same initial velocity as the plane, that velocity disappears as well. In that reference frame:

$$\vec{r}_p = \vec{r}_{p,0}$$

$$\vec{r}_m = \frac{1}{2}\vec{a}_m t^2$$

This immediately tells us how far the missile must travel. It must cover the distance between the two planes, which is 500 m.

$\boxed{\text{Yes}}$ The missile hits the plane.

Now we find the time by setting the position vectors equal.

$$\vec{r}_p = \vec{r}_m$$

$$\vec{r}_{p,0} = \frac{1}{2}\vec{a}_m t^2$$

$$500\ \mathrm{m} = \frac{1}{2}10\ \frac{\mathrm{m}}{\mathrm{s}^2}\mathrm{t}^2$$

$$\boxed{t = 10\ \mathrm{s}}$$