AP/Honors Quiz 8 (Solutions)

Full Name, Period, AP/Honors: Instructor- All

1 The pull of gravity

1. (3 pt, **All**) *The radius of Mars is about* $R_m = 3000$ km *and its density is about* $4\ \frac{\text{g}}{\text{cm}^3}$. *The Earth's radius is* $R_e = 6000$ km *and its density is about* $6\frac{\text{g}}{\text{cm}^3}$. *Which is the closest to the surface gravity of Mars?*

   A. $1\ \frac{\text{m}}{\text{s}^2}$

   $\boxed{\text{B.}}$ $3.5\ \frac{\text{m}}{\text{s}^2}$

   C. $7\ \frac{\text{m}}{\text{s}^2}$

   D. $13.5\ \frac{\text{m}}{\text{s}^2}$

   Use proportionality:

$$g = \frac{GM}{R}$$

$$\frac{g_m}{g_e} = \frac{\frac{\cancel{G}M_m}{R_m^2}}{\frac{\cancel{G}M_e}{R_e^2}} = \left(\frac{M_m}{M_e}\right)\left(\frac{R_e}{R_m}\right)^2$$

Now we note that

$$M = \frac{4}{3}\pi R^3 \rho$$

so

$$\frac{M_m}{M_e} = \left(\frac{R_m}{R_e}\right)^3 \left(\frac{\rho_m}{\rho_e}\right)$$

This means that

$$\frac{g_m}{g_e} = \left(\frac{R_m}{R_e}\right)^3 \left(\frac{\rho_m}{\rho_e}\right)\left(\frac{R_e}{R_m}\right)^2 = \left(\frac{R_m}{R_e}\right)\left(\frac{\rho_m}{\rho_e}\right)$$

Plugging in our numbers

$$\frac{g_m}{g_e} = \left(\frac{1}{2}\right)\left(\frac{4}{6}\right)$$

2. (4 pt **AP**) *Two metal spheres with masses* M_1 *and* M_2 *and radii* R_1 *and* R_2 *are placed far away from each other. The density of both objects is doubled and the radii of both objects are doubled. What happens to the gravitational force between them?*

   A. 8 times larger

   B. 64 times larger

   C. 128 times larger

   $\boxed{\text{D.}}$ 256 times larger

3. (4 pt, **Honors**) *Two spherical objects have the same mass and the same radius. Which of these statements are definitely true?* ***Select all that apply.***

   $\boxed{\text{A.}}$ The average densities are the same.

   $\boxed{\text{B.}}$ The escape speeds are the same.

   C. The periods of rotation are the same.

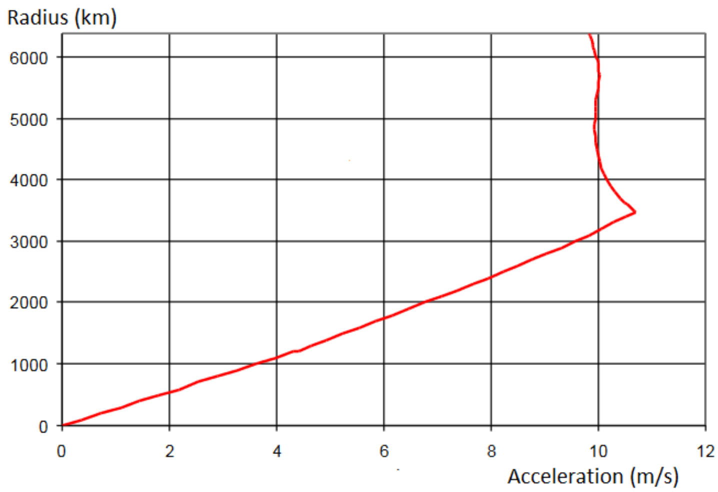
   $\boxed{\text{D.}}$ The surface gravities are the same.

   Explanations:

We look at each of the answers independently and see if we can express the quantity in question as a function of only M and R A: $\rho = \frac{M}{V}$
B: $v_{esc} = \sqrt{\frac{2GM}{R}}$
C: Has nothing to do with this
D. $g = \frac{GM}{R^2}$

2 Plotting stuff

Here is a real graph relating gravitational acceleration and radius inside the earth.



1. (4 pt, **Honors**) *Describe qualitatively how the gravity behaves as we move outward.*

   As we move outward, gravity initially increases linearly until we reach about 3000 km. It then stays relatively constant from 3000 km until we reach the surface.

2. (2 pt, **AP**) Once you exceed 6500 km, how should gravity scale with distance from the planet's center?

   6500km is outside the planet. Outside the planet, mass does not change with radius because density is 0. This means that $\boxed{\mathrm{g} \propto \frac{1}{\mathrm{R}^2}}$

3. (2 pt, **AP**) *Approximately how does density scale with radius for radii between 0 and 3000 km? (You may assume that the graph is a straight line between those points.)*

   We do the same thing as before, but now

   $$g \propto R$$

   this means we have

   $$R \propto \rho R$$

   or

   $$\boxed{\rho \propto R^0}$$

   That is ρ is constant with radius

3 Escaping a Neutron Star

1. (3 pt, **All**) *The escape speed of the sun is about* 600 $\frac{\text{km}}{\text{s}}$ *and the radius of the sun is about* 7×10^5 km. *A neutron star is a star made up almost entirely of neutrons. They have the mass of about twice the mass of the Sun condensed into a radius of* 7 km. *Find the escape speed of a neutron star.*

Proportionality yet again (see a pattern?).
The escape speed is given by

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

I'm going to condense the steps a bit, see the first multiple choice for a more detailed approach.

We know everything except the different numbers will end up canceling, so we know that

$$M_{ns} = 2M_{sun}$$

and

$$R_{ns} = \frac{1}{10^5} R_{sun}$$

Then the escape speed of a neutron star is

$$v_{esc,ns} = \left(\sqrt{2 \times 10^5}\right)\left(600\ \frac{\text{km}}{\text{s}}\right)$$

I was ok with this as an answer, but if you wanted to

$$\sqrt{2 \times 10^5} = \sqrt{2}\sqrt{10}\sqrt{10^4} \approx 1.4 * 3.2 \times 10^2 \approx 450$$

So the escape speed is

$$2 \times 10^5\ \frac{\text{km}}{\text{s}}$$

Or about

$$\boxed{\frac{2}{3}c}$$

2. (2 pt, **All**) *How far do we need to be from the surface of the neutron star for it's escape speed to be the same as the sun?*

   Since the mass is twice as large, we need to be twice as far away.

4 An orbit

1. An object is orbiting the sun (mass $M_\odot$) at a radius R.

   (a) (2 pt, **All**) *What is the velocity of the object?*
   We use centripetal force balance

   $$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

   Solving gives

   $$\boxed{v = \sqrt{\frac{GM}{R}}}$$

   (b) (2 pt, **All**) *By what factor would velocity need to change for the orbit to become unbound?*
   The orbit is unbound if the total energy is 0.

   $$-\frac{GMm}{R} + \frac{1}{2}mv^2 = 0$$

   Plugging in some constant, α times our value of v from above, we get

   $$-\frac{GMm}{R} + \frac{1}{2}\alpha^2\frac{GMm}{R} = 0$$

   Which simplifies to

   $$\frac{GMm}{R}\left(-1 + \frac{1}{2}\alpha^2\right) = 0$$

   $$\alpha = \sqrt{2}$$

   So

   $$\boxed{v_{new} = \sqrt{2}v_{old}}$$