AP Quiz: Mostly rotation

Full Name, Period, AP/Honors:

Select 4 questions to complete. Only honors students may select the honors questions. AP students must select from the remaining questions. If more than 4 questions are completed and it is not clear which should be graded, the worst 4 will be chosen.

1 So many ways to stab myself in the foot!

1. An ice skater begins to spin with her arms straight out. After some time, she pulls her arms in close to her torso. What happens to her angular speed? (The ice is assumed to be frictionless and unable to apply a torque to her.)

   A. It decreases because kinetic energy is conserved and more energy is now in her torso.

   B. It decreases because angular momentum is conserved, and more angular momentum is now in her torso

   C. It increases because kinetic energy is conserved and kinetic energy transferred from her arms to her torso.

   $\boxed{\text{D.}}$ It increases because angular momentum is conserved and the moment of inertia decreased.

   E. It stays the same because both kinetic energy and angular momentum are conserved.

   Explanation: There is no external torque so angular momentum is conserved. Energy is not conserved because the skater uses some of her chemical energy when she pulls her arms in. When the arms come in, moment of inertia decreases because the mass got closer to the axis of rotation. This means that her angular speed increases.

2. You have a solid disk with mass M and radius R that is mounted on a frictionless axle with negligible radius. You apply a constant force F tangent to the edge of the disk and keep applying the force until the disk does exactly 1 full revolution. What will be the final angular speed of the disk?

   A. $\omega = \frac{4\pi F}{MR}$

   $\boxed{\text{B.}}$ $\omega = \sqrt{\frac{8\pi F}{MR}}$

   C. $\omega = \frac{4F}{MR}$

   D. $\omega = \sqrt{\frac{2F}{MR}}$

   E. $\omega = \pi\sqrt{\frac{5F}{MR^2}}$

   Explanation: Use $W = FD$ (you could use angular momentum, but we aren't given time, so it won't be easy). When we rotate one full revolution, a point on the edge goes a distance of $2\pi R$. Now we use $W = \Delta E$ to get

   $$2\pi RF = \frac{1}{2}I\omega^2$$

   We use that the moment of inertia of a disk is $\frac{1}{2}MR^2$ and solve for ω

3. Which of these definitions of x and y could you graph so that the slope of the line would represent the constant A in
$$z^2 = e^{At}$$
   A. $y : (\ln z)^2$ against $x : t$
   $\boxed{\text{B.}}$ $y : 2\ln z$ against $x : t$
   C. $y : At$ against $x : z^2$
   D. $y : e^{2z}$ against $x : e^{At}$
   E. $y : z^2$ against $x : e^t$
   Explanation: Start with $z^2 = e^{At}$. Now take ln of both sides
$$\ln(z^2) = At$$
   Then we can use log rules: $\ln(z^2) = 2\ln(z)$

4. (**Honors only**) A rod that is free to rotate and translate but is initially static is struck at its end with a ball (mass m) that had an initial velocity v_b. The ball recoils with a speed $-v_{ball}$. What will be the final momentum of the rod?
   A. $p = -2mv_b$
   $\boxed{\text{B.}}$ $p = 2mv_b$
   C. $p = mv_b$
   D. $p = \frac{mv_b}{3}$
   E. $p = \sqrt{2mv_b + v_b^2}$
   Explanation: Ignore the rotation of the object, that doesn't involve linear momentum. Then we just have that the ball changed momentum from mv_b to $-mv_b$ (a change of $-2mv_b$), which means that the rod had to change from 0 to $2mv_b$

5. (**Honors only**) Which of these definitions of x and y could you graph so that the slope of the line would represent the constant A in
$$z = At^3$$
   A. $y : z$ against $x : t$
   B. $y : z$ against $x : t^{1/3}$
   C. $y : At^2$ against $x : z$
   $\boxed{\text{D.}}$ $y : z$ against $x : t^3$
   E. $y : z^2$ against $x : t^{5/2}$

2 Freedom to respond as indicated

1. *A vertically oriented rod that is free to rotate and translate but is initially static is struck at its end with a ball (mass m) that had an initial horizontal velocity $v_b\hat{\mathrm{x}}$. The ball recoils with a velocity nearly $-v_b\hat{\mathrm{x}}$. The rod ends up with a linear speed v and an angular speed ω. What was the length of the rod? The moment of inertia of a rod with total length R, pinned in the center, is $\frac{1}{12}MR^2$.*

   Both linear and angular momentum must be conserved. We'll start with angular.

$$-\Delta\vec{L}_b = \Delta\vec{L}_r$$

$$2mv_b\frac{L}{2} = \frac{1}{12}M_rL^2\omega$$

Now lets look at linear

$$-\Delta\vec{p}_b = \Delta\vec{p}_r$$

$$2mv_b = M_rv_r$$

but $2mv_b$ appears exactly in the angular momentum equation, so let's substitute that for M_rv_r, and the angular momentum equation becomes

$$M_rv_r\frac{L}{2} = \frac{1}{12}M_rL^2\omega$$

This can now be solved to give

$$\boxed{L = 6\frac{v_r}{\omega}}$$

2. Two disks have their centers on the same axis. The disks initially have

   - Disk 1
     - Mass M
     - Radius R
     - Angular velocity ω clockwise
   - Disk 2
     - Mass M
     - Radius $R/2$
     - Angular velocity ω counterclockwise

   After the two disks come together and begin to rotate as a single object, what will be the final angular velocity?

   We got straight into angular momentum conservation, they will stick at the end, so the moments of inertia add there

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$$

   Note that ω_1 and ω_2 are in opposite directions, so we need to subtract them.

$$I_1\omega - I_2\omega = (I_1 + I_2)\omega_f$$

   Now we solve for I_2 in terms of I_1

$$I_1 = \epsilon M R^2$$

$$I_2 = \epsilon M \frac{R^2}{4} = \frac{1}{4} I_1$$

   Then our angular momentum equation becomes

$$I_1(1 - \frac{1}{4})\omega = I_1(1 + \frac{1}{4})\omega_f$$

$$\boxed{\omega_{\mathrm{f}} = \frac{3}{5}\omega}$$