Gravity Homework, Part II

Full Name, Period: **due: x/xx**

1. A planet of radius R has uniform density ρ, except that a spherical cave of radius a has been hollowed out of it. The cave is not centered on the planet: its center sits at a position $\vec{d}$ from the planet's center, with $d + a < R$. Do not attempt to integrate anything in this problem.

   (a) Find the gravitational field at an arbitrary point inside the cave.

   (b) Describe your answer in words. Is anything about it surprising?

2. A moon of radius r and density ρ_m orbits a planet of radius R_p and density ρ_p, at a distance d between their centers. Get too close and the planet's gravity rips the moon apart. We can estimate when with a scaling argument.

   (a) Consider the two points on the moon nearest to and farthest from the planet. Show that the difference between the planet's gravitational field at those two points is approximately $\frac{4GM_p r}{d^3}$, where M_p is the planet's mass. You will want to expand for $r \ll d$.

   (b) The moon holds itself together with its own gravity. Estimate the field at the moon's surface due to the moon itself, and find the separation d at which the stretching just overcomes it.

   (c) Rewrite your answer in terms of the two densities and the planet's radius. What does it say if the two objects have the same density?

3. Before nuclear physics existed, the best available guess was that the Sun shines by slowly contracting, converting gravitational potential energy into heat and light. Let us see how well that works. Take the Sun to be a uniform sphere with $M_\odot = 1.99 \times 10^{30}$ kg and $R_\odot = 6.96 \times 10^{8}$ m, radiating at $L_\odot = 3.83 \times 10^{26}$ W.

   (a) Find the gravitational potential energy of a uniform sphere of mass M and radius R, by imagining it assembled shell by shell from material brought in from infinity.

   (b) How long could the Sun shine at its present brightness on that energy alone?

   (c) The Earth is about 4.5×10^{9} years old and there is fossil evidence of life for most of that time. What does your answer imply, and what is the resolution?

4. Our own galaxy has a flat rotation curve, like the one on the previous homework. The Sun sits about 8 kpc from the galactic center and orbits at roughly 220 km/s. One parsec is 3.09×10^{16} m, and $M_\odot = 1.99 \times 10^{30}$ kg.

   (a) Estimate the mass of the galaxy interior to the Sun's orbit, in solar masses.

   (b) The flat rotation curve continues out to at least 50 kpc. Estimate the mass interior to that radius. What fraction of it lies inside the Sun's orbit?

   (c) Suppose the flat rotation curve continued forever. What would that imply about the total mass of the galaxy? Is that acceptable, and what must actually happen?

5. A science fiction novel describes a hollow planet: a thick spherical shell of total mass M with an empty region of radius a inside it, where the characters live standing on the inner surface with the sky arching overhead.

   (a) What is the gravitational field everywhere inside the hollow region? What does this mean for the characters?

   (b) Would making the shell more massive, or much thicker, change your answer?

   (c) Suppose the shell were not quite spherical, say slightly egg-shaped. Would the field inside still vanish? You do not need to calculate anything.