Forces and Kinematics: Extra Practice

Full Name, Period:

These are for practice and are not collected. Every answer should be in symbols.

1. An object of mass m is released from rest and falls under gravity through air that exerts a drag force $\vec{F}_d = -b\vec{v}$, where $b > 0$.

   (a) Without solving anything, find the terminal speed v_T and the only combination of m, b and g that has units of time. Call it τ.

   (b) Find $v(t)$, taking downward as positive.

   (c) Find $x(t)$, measuring x downward from the release point.

   (d) Show that for $t \ll \tau$ your answers reduce to the familiar free-fall results, and explain physically why they must.

2. A puck of mass m slides on frictionless ice with initial speed v_0. The only horizontal force is air resistance. Consider two models for it: a linear drag $F = -bv$ and a quadratic drag $F = -cv^2$.

   (a) For each model, find $v(t)$.

(b) For each model, find the total distance traveled before the puck stops. Comment on the comparison.

(c) For the quadratic model, find $v(x)$ directly, without using your answer to (a).

3. A uniform rope has mass M and length L.

   (a) The rope hangs at rest from a ceiling. Find the tension as a function of the distance y measured upward from the bottom end.

   (b) The rope now lies on a frictionless floor and is pulled by a horizontal force F applied to one end. Find the tension a distance x from the pulled end.

   (c) A block of mass M_b is now tied to the far end of that rope, and F is applied as before. Find the tension at both ends of the rope, and show that the massless-rope result is recovered in the appropriate limit.

4. A block of mass m rests on the frictionless inclined face of a wedge of mass M, which sits on a frictionless floor. The incline makes an angle θ with the horizontal. A horizontal force F is applied to the wedge.

   (a) Find the F for which the block does not slide relative to the wedge.

   (b) Redo the argument in the reference frame of the wedge, and state the condition in a single sentence about "effective gravity."

   (c) Check the limits $\theta \to 0$ and $\theta \to \frac{\pi}{2}$, and say in words what each is telling you.

5. A curve of radius R is banked at an angle θ. The coefficient of static friction between the tires and the road is μ.

   (a) Find the one speed v_0 at which the curve can be taken with no friction at all.

   (b) Find the maximum speed for which the car does not slide.

   (c) Find the minimum speed, and the condition on μ and θ under which there is no minimum at all.

6. A ball of mass m is swung in a vertical circle of radius L.

   (a) The ball is on the end of a string. Find the minimum speed it can have at the top of the circle, and explain what physically goes wrong below that speed.

   (b) The string is replaced by a rigid rod of negligible mass. How does the answer change, and why?

(c) With the string at its minimum speed at the top, find the speed and the tension at the bottom.

(d) Show that $T_{bot} - T_{top}$ has the same value for *any* speed, not just the minimum one.

7. A projectile is launched with speed v_0 from the bottom of a hill whose surface is a straight slope rising at angle ϕ. The launch angle is θ, measured from the horizontal.

   (a) Find the range R measured along the slope.

   (b) Find the launch angle that maximizes R, and the maximum range itself. Interpret the angle geometrically.

8. A uniform chain of mass M and length L lies on a frictionless table with a length x_0 hanging over the edge. It is released from rest. Take x to be the length hanging over the edge, and assume the chain stays taut and the corner does not impede it.

   (a) Show that $\ddot{x} = \frac{g}{L}x$, and comment on how this compares to the mass-on-a-spring equation.

(b) Solve for $x(t)$ with the given initial conditions.

(c) Find the time at which the chain completely leaves the table.

(d) Find the speed of the chain at that moment, and check your answer by an independent method.

9. A block is launched up a rough incline of angle θ with initial speed v_0. The coefficient of kinetic friction is μ_k. Three students propose expressions for the distance d measured along the incline that the block travels before coming momentarily to rest:

   - Student 1: $d = \dfrac{v_0^2}{2g\,(\sin\theta + \mu_k\cos\theta)}$
   - Student 2: $d = \dfrac{v_0^2}{2g\,(\sin\theta - \mu_k\cos\theta)}$
   - Student 3: $d = \dfrac{\mu_k v_0^2}{2g\sin\theta\cos\theta}$

   (a) Without solving the problem, determine which of these can be rejected, and give the specific reason in each case.

(b) Now derive the correct expression.

(c) Does the block slide back down? State the condition, and explain why it involves a different coefficient than the one in part (b).