Energy Homework, Part II

Full Name, Period: **due: x/xx**

1. A particle moves in the plane under the force $\vec{F} = k\,(y\hat{x} - x\hat{y})$.

   (a) Find the work done in moving the particle from the origin to the point (a,a) along the path that goes first along the x axis to $(a,0)$, then straight up to (a,a).

   (b) Now find the work along the path that goes first up the y axis to $(0,a)$, then straight across to (a,a).

   (c) Could you define a potential energy for this field? If so, define it. If not, explain why it isn't possible.

2. A particle of mass m moves along the x axis in the potential energy

$$U(x) = U_0\left[\left(\frac{x}{a}\right)^4 - 4\left(\frac{x}{a}\right)^2\right]$$

   where U_0 and a are positive constants.

   (a) Find all the equilibrium positions and state which are stable and which are unstable.

   (b) The particle is released from rest at $x = a$. Describe the resulting motion, and find the positions where it momentarily comes to rest.

   (c) Find the minimum kinetic energy the particle would need, at the bottom of the right-hand well, to reach the left-hand well.

   (d) Can the particle escape to $x = \infty$ if given enough energy? Explain.

3. A uniform chain of total mass M and total length L lies on a frictionless table, with a length ℓ hanging over the edge. The chain has linear density $\lambda = \frac{M}{L}$.

   (a) Find the work you must do to pull the hanging part back onto the table, by integrating the force you apply.

   (b) Now find the same work using the center of mass of the hanging piece, and confirm that the two answers agree.

   (c) The chain is released from rest with the length ℓ hanging. Find its speed at the moment the last link leaves the table.

4. A car of mass m starts from rest and its engine delivers a constant power P. Ignore all friction and drag.

   (a) Find the speed as a function of time.

   (b) Find the distance traveled as a function of time.

5. A planet of mass M and radius R has uniform density. A narrow tunnel is drilled straight through its center, and a ball of mass m is dropped from rest at the surface. Neglect all friction and air resistance.

   (a) Show that the gravitational force on the ball while it is inside the planet is $\vec{F} = -\frac{GMm}{R^3} r\ \hat{\mathrm{r}}$, and use this to find the potential energy inside.

   (b) Find the speed of the ball as it passes through the center.