Statics

Full Name, Period: **due: x/xx**

1. You want to hang a an extremely long rope vertically. The rope is to be so long that you are concerned that the rope won't be able to hold up it's own weight. The rope has a density of ρ and can support a maximum force/area of σ, You may assume that the rope is not so long that gravity differs meaningfully from g.

   (a) Let $M(z)$ be the total mass of the rope up to height z. Find the expression for $M(z)$ that maximizes the potential rope length. There will be an arbitrary constant, call it M_0.

   (b) Determine the height required for the rope's mass to change by a factor of 2.

   (c) Assume that we want to support an object of mass m from the end of the rope. If the rope is to have a length l, determine the mass of the rope.

   (d) Plug in values appropriate to steel wire $\rho = 8000\ \frac{\text{kg}}{\text{m}^3}$, $\sigma = 400$ MPa, that needs to support a mass of 10 kg at the end. If the attachment point at the top of the wire can support 1000 N, how long can the cable be?

2. You want to lift a long beam vertically with 2 wires spaced some distance apart. Should they be far apart, or close together and why? Assume that they are always symmetric around the midpoint.

   A. Far apart. This ensures better stability in case one wire is ever slightly longer than the other.

   B. Far apart. This allows more torque on the bar which aids in lifting it.

   C. Close together. When placed very far (10s or meters) apart, the torques can get large enough to snap the wires.

   D. Close together. Putting the wires near the center of mass ensures gravity cannot apply a torque on the system.

3. You are trying to lift an object with a mass of M. You have unlimited real pulleys, which each add frictional force $F_f = \alpha F + F_0$ where F is the normal force on the axle, α is a dimensionless constant, and F_0 is a constant force.

   (a) Explain why a single pulley isn't useful for multiplying force.

   (b) Provide a physically plausible reason why adding more pulleys will eventually provide minimal help.

   (c) Show that in the limit that you have infinite pulleys, your required force approaches a constant (non-zero) value.

   (d) Interpret the value of the limit you took. Does this agree with the intuition you provided?

   (e) The efficiency of a machine is defined as $\epsilon = \frac{W}{E}$ where W is the work done by the machine, and E is the energy input. What is the efficiency of this machine in the infinite pulley limit?

4. A manufacturer claims that their new wrench can provide 1000 N$\cdot$m of torque on a nut before slipping. You are suspicious that the actual maximum is far less than this. Provide a procedure to test the claim. You may assume that you have a nut and bolt capable of handling the torque.