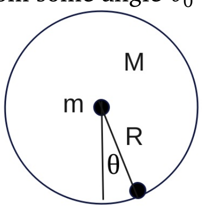
Rotation Homework

Full Name, Period: **due: x/xx**

1 Imagination Land Problems

1. Consider a thin rod with length $l = 10$ m, density $1000\ \frac{\text{kg}}{\text{m}^3}$ and radius 0.1 m. Attached to the ends are spheres of radius 1 m and 2m that have their centers at -6 m and 7 m respectively. Both spheres have density $1000\ \frac{\text{kg}}{\text{m}^3}$. The rod is released from $\theta_0 = 0.1$ rad. Find the maximum angular speed of the rod.

2. You create a physical pendulum by gluing a hockey puck of mass $m << M$ and radius $r << R$ to the edge of a large disk of mass M and radius R as shown in the diagram. The large disk is pinned in its center, released from some angle θ_0 and allowed to oscillate.

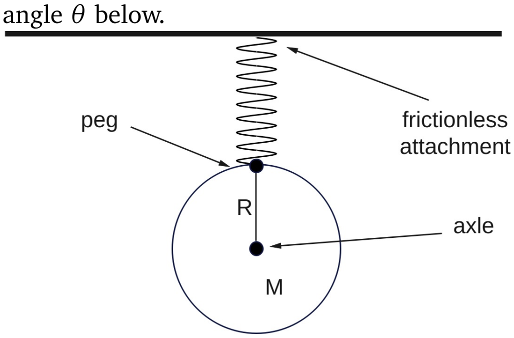


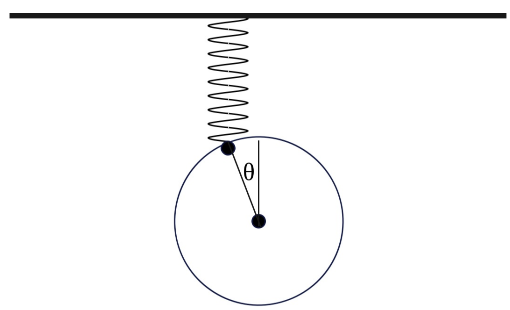
   (a) Find the oscillation period assuming that $\theta_0 << \frac{\pi}{2}$. Hint, use $\sin\theta \approx \theta$

   (b) Sketch a plot of θ as a function of time.

   (c) Find the minimum force that the glue must be able to apply for the hockey puck to stay on.

3. A device is created that consists of a spring (constant $k = 10\ \frac{\text{N}}{\text{m}}$) with one end attached to a frictionless rod and the other end attached to a peg on a uniform disk with mass $M = 1$ kg. The frictionless rod will allow the spring to always remain parallel to its original configuration. The spring is in equilibrium at the start. Call the current rotation angle θ. See diagrams of the equilibrium of the system and the system at some angle θ below.





   (a) Find a differential equation for the angle θ. You should probably leave things in terms of variables here.

   (b) Assuming you start from an angle of $\frac{\pi}{2}$, use a numerical solver to find the solution to the equation and plot it. Sketch the plot and estimate the period from the plot.

   (c) Repeat the above for a few very small angles. Notice that the period changed each time.

   (d) In the event that the starting angle $\theta_0 << \frac{\pi}{2}$, simplify your differential equation by replacing trig functions with small angle approximations. Use this approximation to explain why period is not independent of the amplitude, even for small angles.

   (e) Use a numerical solver to find the solution to the simplified equation for and initial angle of 0.1 rad (or use a CAS to find and plot the exact solution to the simplified equation) and compare the period to the original equation. How large must the angle get before they differ substantially?

4. Most wheels are mounted on bearings, which have a friction that is dependent on rotation speed. We can make a (vastly simplified so that it has an analytic solution) model of this using $\tau_f = \gamma\omega$ (opposing motion). The wheel has a moment of inertia ϵMR^2.

   (a) The wheel is spinning with an initial angular speed ω_0. Without solving for ω explicitly, sketch a graph of the angular speed vs time. You do not need to label anything.

   (b) Create and solve a differential equation for ω. If you plug some numbers in, does your equation produce the graph you predicted?

   (c) You apply a time dependent force $F = \beta t$ to the edge of the wheel. Without solving for ω explicitly, sketch a plot of ω vs t in this case. You do not need to label anything.

   (d) Create a differential equation for ω and solve it with a calculator. If you plug numbers in and plot, does it produce the graph you predicted?

2 Real World Problems

1. Until the last century or so, the most accurate timekeeping devices humans possessed were pendulum clocks. They worked by having a disk shaped pendulum bob (radius r) at the end of a rigid rod (length l). Take the rod to have mass M_r and the pendulum bob to have mass M_p. Assume the rod and disk are uniform.

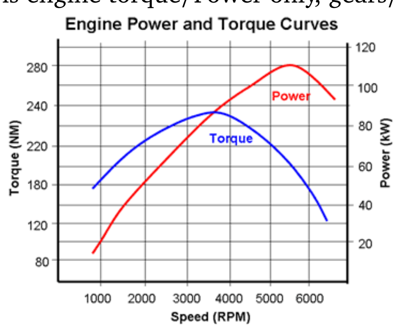
   (a) If we want to maximize the accuracy of the pendulum, should a very dense material or a low density material be used? Why? Do NOT neglect any relevant effects.

   (b) Now neglect air resistance. Is the period of the pendulum independent of M_p? Explain your answer without manipulating equations.

   (c) Derive a formula that relates the period of the pendulum to any relevant quantities in the problem along with constants. You may assume small amplitude oscillations.

   (d) Show that your expression produces the behavior you suggested in part b.

2. Internal combustion engines (ICE) are typically not capable of achieving a large torque at low RPM. Unless special methods are used (which tend to be destructive to the car), this limits the maximum acceleration of ICE cars. Below are torque and Power curves for a car, (image credit Horizon Technologies). Note that this is engine torque/Power only, gears/wheels are not included.



   (a) Conceptually, why would the power peak after the torque?

   (b) If you check the intersection of the torque and power curves, they intersect at just over 3500 RPM. The Hellcat is an extremely high performance sports car. If we checked another vehicle, what would we find about the RPM that the intersection occurs at assuming units are kept the same.

      A. It would likely be substantially lower.

      B. It would likely be substantially higher.

      C. It could be substantially lower, or substantially higher

D. It would be similar, but not likely the same

E. It would be exactly the same

(c) Justify your answer. Your justification may involve equations, but should include 1-2 sentences of explanation.