Math Prerequisites Homework

Full Name, Period: **due: x/xx**

1. Sketch a vector field $\vec{v}$ that would have the property that $\oint_C \vec{v} \cdot d\vec{l} = 0$ for all closed curves that do not pass through the origin.
2. Sketch a vector field $\vec{v}$ that would have the property that $\oint_S \vec{v} \cdot d\vec{A} = 0$ for all finite closed surfaces.
3. Is it possible to create a vector field with a zero divergence and zero curl everywhere that is not identically $\vec{0}$ everywhere? Provide an example, or explain why it is impossible.
4. Find the flux of the vector field given by $k\sqrt{x^2 + y^2}\hat{z}$ through the circle with radius R and unit normal $\hat{n} = \hat{z}$ centered at the origin. Hint: cylindrical coordinates are your friend.
5. Find the flux of the vector field given (in spherical coordinates) by $kr\hat{r}$ through the circle with radius R and unit normal $\hat{n}$ centered at the point $z = a$. Hint: cylindrical coordinates are still your friend if you get clever with the dot product.
6. You have the conservative vector field $\vec{v} = \frac{k}{r^2}\hat{r}$. Find the value of $\int_C \vec{v} \cdot d\vec{l}$ along some arbitrary curve C that begins at infinity and ends at $(x_0, 0, 0)$
7. Find the mass of the cylindrical shell, with length l oriented along $\hat{z}$ with inner radius a and outer radius b whose density function is $\rho(\vec{r}) = k\sqrt{x^2 + y^2}$. There are no top or bottom faces.
8. (optional) Consider the 4D hypercube with side length L centered at the origin. Its density function is given by $\rho(w, x, y, z) = k|wxyz|$. Find the mass of the cube. Hint: exploit symmetry.